Homework -6

3.3.5)

$$g(x) = \begin{cases} 0 & , 0 < x < 0.4a \\ 5c & , 0.4a < x < 0.6 \\ 0 & , 0.6a < x < a \end{cases}$$

$$G(x) = \begin{cases} 0 & 0 < x < 0.4a \\ f(x) & 0.4a < x < 0.6 \\ a & 0.6 < x < a \end{cases}$$

$$G(x) = \frac{1}{c} \int_{0.4a}^{x} 5c\,dy = 5[x - 0.4a)$$

At $x = 0.6a$

$$G(x) = 5[0.6a - 0.4a] = a.$$

$$\therefore G(x) = \begin{cases} 0 & 0 < x < 0.4a \\ 5(x - 0.4a) & 0.4a < x < 0.6a \\ a & 0.6a < x < a \end{cases}$$

3.6.6)

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \qquad t > 0$$

$$u(x, 0) = 0 \quad , x > 0$$

$$\frac{\partial u}{\partial t}(x, 0) = 0 \qquad x > 0$$

$$u(0, t) = h(t) \qquad t > 0$$

$$h(t) = \begin{cases} 0 & 0 < t < 1 \\ 1 & 1 < t < 2 \\ 0 & 2 < t \end{cases}$$

$$u(x, t) = \phi(x - ct)$$

$$u(0, t) = h(t) \Rightarrow \phi(-t)$$

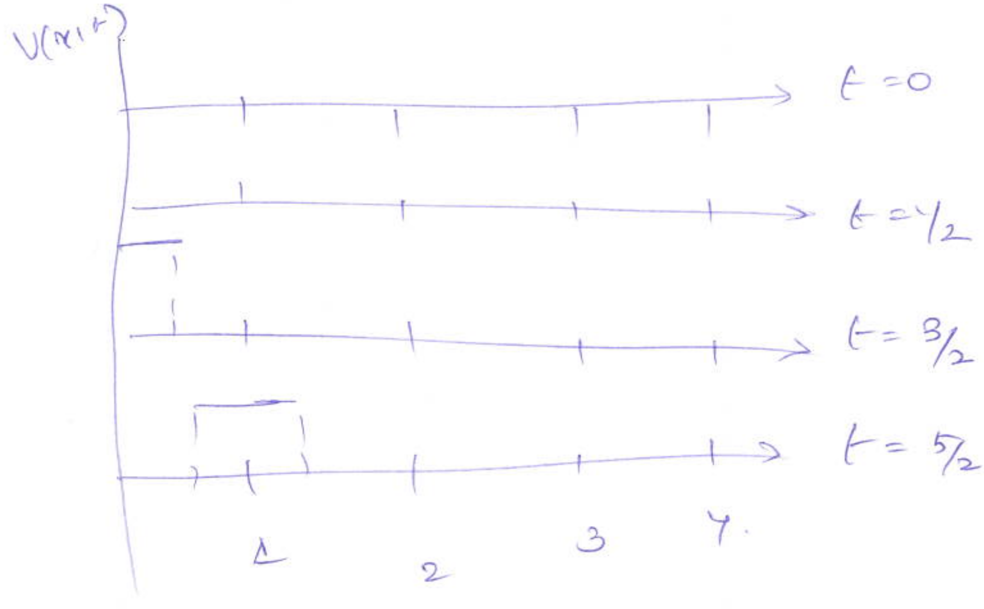
$$\phi(q) = \begin{cases} 0 & q > 0 \\ h(-q/c) & q < 0 \end{cases}$$

$$t = 0 \Rightarrow u(0, 0) = 0$$

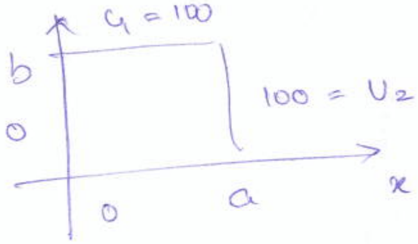
$$t = -1/2 \Rightarrow u(0, 1/2) = \phi(-1/2)$$

$$t = 3/2 \quad u(0, 3/2) = \phi(-3/2)$$

$$t = 5/2 \quad u(0, 5/2) = h(5/2)$$



4.2.7b)



$$U(x,y) = U_1(x,y) + U_2(x,y)$$

$$U_1(x,y) \Rightarrow \quad a_n = \frac{2}{a}\int_0^a 0 \; = 0$$

$$C_n = \frac{200}{n\pi}\left[1-\cos n\pi\right]$$

$$b_n = \frac{C_n - a_n \cosh(\lambda_n b)}{\sinh(\lambda_n b)}$$

(or) $$b_n = \frac{200}{n\pi \sinh\left(\frac{n\pi}{a} b\right)}\left\{1-\cos(n\pi)\right\}$$

$$U_1(x,y) = \sum_{n=1}^{\infty} \frac{200}{n\pi \sinh\left(\frac{n\pi b}{a}\right)}\left(1-\cos n\pi\right)\Big\} \sinh\left(\frac{n\pi}{a}y\right)\sin\left(\frac{n\pi}{a}x\right)$$

$$A_n = \frac{2}{b}\int_0^b g_2(y)\sin(\mu_n y)\,dy$$

$$A_n = \frac{200}{n\pi}\left[1-\cos n\pi\right]$$

$$B_n = 0$$

$$\therefore \; U_2(x,y) = \sum_{n=1}^{\infty} \frac{200}{n\pi \sinh\left(\frac{n\pi}{b}a\right)}\left(1-\cos n\pi\right)\sinh\left(\frac{n\pi}{b}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

4.3.2 (b) $U(x,b) = 100$; $u(0,y) = 0$; $u(a,y) = 100$; $\frac{\partial u}{\partial y}(x,0) = 0$

General Solution:

$$U(x,y) = \sum (a_n \cosh(\lambda_n y) + b_n \sinh(\lambda_n y)) \sin(\lambda_n x)$$

$$U_y(x,0) = \sum \lambda_n b_n \sin(\lambda_n x) = 0 \Rightarrow b_n = 0$$

$$U(x,b) = \sum a_n \cosh(\lambda_n b) \sin(\lambda_n x) = 100$$

$$a_n \cosh(\lambda_n b) = \frac{2}{a} \int_0^a 100 \sin(\lambda_n x)\, dx.$$

$$a_n = \frac{\frac{200}{n\pi}\left[1 - \cos n\pi\right]}{\cosh(\lambda_n b)}$$

$$U_2(x,y) = \sum \cos(\mu_n y)\, A_n \cosh(\mu_n x) + B_n \sinh(\mu_n x))$$

$$U_2(0,y) = 0 = \sum \cos(\mu_n y) A_n \Rightarrow A_n = 0$$

$$U_2(a,y) = 100 = \sum \cos(\mu_n y) \cdot B_n \sinh(\mu_n a)$$

$$B_n = \frac{200(-1)^n}{(n - 1/2)\pi} \cdot \frac{1}{\sinh(\mu_n a)}$$

$$U(x,y) = \sum \left\{ \frac{200}{n\pi}\left[1 - \cos n\pi\right] \frac{\cos \lambda_n y \sin \lambda_n x}{\cos \lambda_n b} + \right.$$

$$\frac{200(-1)^n}{(n - 1/2)\pi} \frac{\sinh(\mu_n x)}{\sinh(\mu_n a)} \cos(\mu_n y) \qquad \lambda_n = \frac{n\pi}{a}$$

$$\mu_n = (n - 1/2)\frac{\pi}{b}$$

4.4.15

$$\begin{cases} \dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} = 0 & -\infty < x < \infty \quad 0 \le y < \infty \\ \end{cases}$$

$U(x,y)$ should be bounded as $y \to \infty$

$$X''Y + Y''X = 0 \Rightarrow \frac{X''}{X} = \frac{-Y''}{Y} = \text{Const} = -\lambda^2$$

$$X'' + \lambda^2 X = 0$$

$$U(x,y) = \frac{1}{\pi} \int_0^\infty \left(A(\lambda)\cos(\lambda x) + B(\lambda)\sin(\lambda x) \right) e^{-\lambda y} \, d\lambda$$

where $\lambda > 0$

$$A(\lambda) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \cos(\lambda x) \, dx$$

$$B(\lambda) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \sin(\lambda x) \, dx.$$