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Proof by Contradiction

of a 2×2 matrix are

$\exists \lambda_1 \neq \lambda_2$ distinct, and assume the corresponding real eigenvectors are not distinct

Assume $e_1 = a e_2$ $\lambda_1 = \lambda_2$

$$\Rightarrow A e_1 = \lambda_1 e_1 \text{ and } A e_2 = \lambda_2 e_2$$

with $e_1 = a e_2$

$$\Rightarrow \lambda_1 e_1 = A e_1 = A (a e_2) = a A e_2 = a \lambda_2 e_2 = \lambda_2 e_1$$

$\therefore \lambda_1 = \lambda_2$ which is impossible because λ_1 and λ_2 are distinct

$\Rightarrow \lambda_1 \neq \lambda_2$ are distinct implies the corresponding real eigenvectors are distinct too

3 points
proof works

2 points
convoluted or
small hole

1 point
really convoluted
or really big hole

0 points
no work or
unfinished